*Journal of Applied Mechanics and Technical Physics, Vol. 42, No. 4, pp. 616–620, 2001*

# HARTMANN EFFECT. REGION OF EXISTENCE AND OSCILLATION FREQUENCIES

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UDC 534.232

*Data of experiments on determining the region of existence for auto-oscillations (Hartmann effect) with a frequency of approximately* 450 Hz *with variation in the distance from the nozzle to the resonator. Results of these experiments differ from the well-known results of Hartmann and his followers obtained for shallow resonators. It is shown that the region of auto-oscillations exists for large distances between the nozzle and the resonator. The results obtained are explained using the modern knowledge of the gas-dynamic structure of a supersonic underexpanded jet. It is shown that in determining the frequency of low-frequency oscillations, it suffices to allow for the resonator length and its "added" mass.*

It is known that auto-oscillations that arise when a supersonic underexpanded jet flows into a half-closed tube (Hartmann effect) exist for a particular range of distances from the nozzle to the resonator $l$ (Fig. 1), called the nozzle–resonator gap or the tuning parameter of a Hartmann generator [1]. In Fig. 1, $x_{\rm m}$ is the distance from the nozzle to the Mach disk in the presence of a resonator, $h$ is the depth of the resonator, $d_1$ is the diameter of the nozzle exit section, $d_2$ is the diameter of the resonator, and $d_{\rm obst}$ is the diameter of the obstacle.

We consider briefly the gas-dynamic and geometrical structure of the jet, which is necessary for analysis of the results obtained. We denote the off-design parameter of the jet by $n = p_{\rm atm}/p_{\rm amb} > 1$ ($p_{\rm atm}$ is the nozzle exit pressure and $p_{\rm amb}$ is the ambient pressure). A supersonic underexpanded jet has a barrel-shaped quasiperiodic structure with a typical particular system of shock waves in the first "barrel" (Fig. 2) [2]. The length of the first "barrel" $L$ is defined by the empirical formula [3]

$$L/r_1 = 1.72 \mathrm{M}_{\rm atm} \sqrt{\gamma}\, n, \qquad (1)$$

where $\mathrm{M}_{\rm atm}$ is the Mach number at the nozzle exit section, $r_1$ is the nozzle radius, and $\gamma$ is the adiabatic exponent. According to [4], the distance from the nozzle exit to the Mach disk (direct shock) in the free jet is $x_{\rm sh} = 0.8L$ (Fig. 2). The position of the jet cross section of maximum diameter is defined by the coordinate $x_{\max}$, which is equal to [5]

$$x_{\max} = 0.8 x_{\rm sh} = 0.64L. \qquad (2)$$

The flow in the second and subsequent "barrels," whose lengths decrease monotonically only slightly compared to the length of the first "barrel," is characterized by the presence of regularly interacting oblique shock waves. The point of intersection of these waves is located on the jet axis (in the middle of the "barrel), i.e., in the second and subsequent "barrels: $x_{\max} = 0.5L$.

The resonator is a semiopen tube (we shall consider only an axisymmetric jet and a cylindrical resonator) which presents its open end to the nozzle and is aligned with it (see Fig. 1). It is known that the resonator placed in the jet "works" as a one-dimensional waveguide for $h \gg d_2$ (see Fig. 1) or as a concentrated vibratory system (Helmholtz resonator) for $d_2 \sim h$. Therefore, one can treat the incident jet as a source of vibrational energy in a one-dimensional approximation, i.e., take into account only the axial distribution of gas-dynamic characteristics averaged over the jet cross section. Averaging is performed by the rule of [6] with allowance for conservation of

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 0021-8944/01/4204-0616 $25.00 

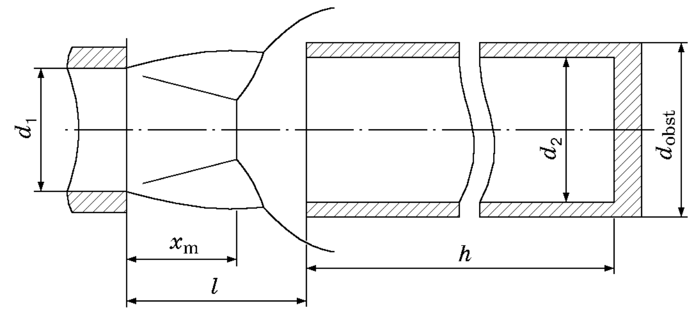


Fig. 1

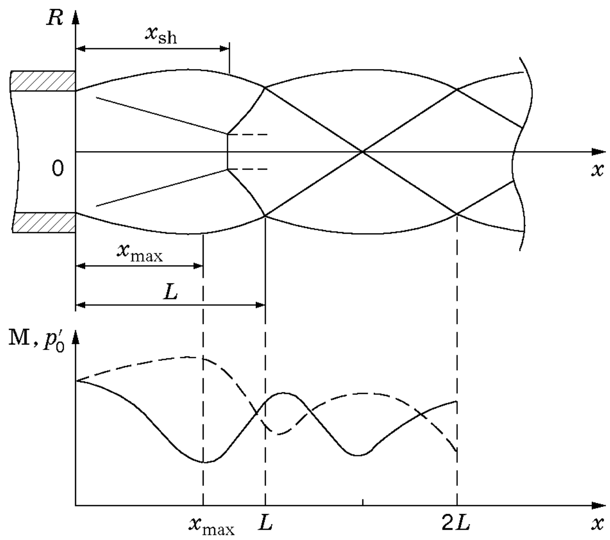


Fig. 2

energy and momentum in the real and averaged flows. As a one-dimensional gas-dynamic object, the jet with specified $M_{atm}$, $n$, $\gamma$, and stagnation temperature $T_0$ is characterized by the axial distribution of the Mach number averaged over the cross section $M(x)$ (dashed curve in Fig. 2). An analysis of the known properties of supersonic flows and the barrel-shaped jet boundary shows that in the first "barrel" of the jet, $M(x)$ increases to the coordinate $x_{max}$, which is determined from Eqs. (1) and (2) and then decreases. For the second and subsequent "barrels," the same dependence holds but $x_{max} \approx 0.5L$. According to this, the distribution of the cross-section-average stagnation pressure past the normal shock $p_0'(x)$ (i.e., the pressure measured at the resonator bottom) is shown qualitatively in Fig. 2 (solid curve). When the resonator is placed in the jet at a distance $l < L$ from the nozzle (see Fig. 1), the system of shock waves, consisting of a Mach disk and a reflected oblique shock, typical of the free jet, moves to the nozzle and takes the new position $x_m$, as in the case of a bluff solid body placed in a homogeneous supersonic flow (similarly to a detached shock).

In [1] and early studies of the region of existence of auto-oscillation, it was assumed that the gas-dynamic structure of the entire jet beginning with the first "barrel" is similar to that shown in Fig. 2 for the second "barrel" (see [1, p. 12]). The static pressure along the jet axis was similar to the curve of $p_0'(x)$ for the second "barrel" given in Fig. 2. It was argued that auto-oscillations exist in the region of $0.5L < l < L$. The displacement of the system of shock waves typical of the free jet toward the nozzle with placement of the resonator in the jet was ignored.

Some properties of auto-oscillations were studied in [6–9]. Thus, Ugryumov [6] determined experimentally the regions of various flow regimes, gave empirical formulas for calculating the beginning and end of the region of intense pulsations for particular geometrical ratios (resonator diameter is equal to nozzle diameter), and analyzed the shock-wave pattern of the flow. As follows from the given oscillogram of the mean flow pressure for $M_{atm} = 2$ and $n = 2.5$, the first region of pulsations is in the range of $0.8L \leqslant l \leqslant 1.76L$ (the beginning of pulsations is conditionally determined by the position of the Mach disk in the first "barrel" of the free jet).

TABLE 1

| $M_{atm}$ | $n$ | $d_1$, mm | $d_{obst}/d_1$ | $A$ | $B$ | $l_{min}/L$ | | $l_{max}/L$ | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | calculation | experiment | calculation | experiment |
| 1.0 | 2.1 | 10 | 1.8 | 1.71 | 2.30 | 0.75 | 0.83 | 1.19 | 1.53 |
| 1.0 | 2.1 | 13 | 1.92 | 1.69 | 2.27 | 0.75 | 0.91 | 1.18 | 1.41 |
| 1.0 | 2.1 | 17 | 1.47 | 1.78 | 2.39 | 0.72 | 0.98 | 1.19 | 1.39 |
| 2.0 | 1.5 | 40 | 1.0 | 1.92 | 2.56 | 0.70 | 0.64 | 1.20 | 1.24 |

From [7, p. 59, Fig. 1] it follows that for $M_{atm} = 2$, $n = 3$, $h/r_1 = 2$, and $d_2/d_1 = 2$, the region of auto-oscillations is in the range of $0.6L < l < 1.5L$. Kuptsov et al. [8] studied auto-oscillations for deep cavities for $M_{atm} = 3.2$–$4.0$, $d_2/d_1 = 0.5$–$2.0$, and $h/d_1 = 9$–$76$. In [8], it is argued that the extent of auto-oscillation regions coincides with the length of “barrels” in a supersonic jet but it is difficult to obtain numerical values because the nondimensional parameter (distance from the nozzle to the resonator) is not determined. Semiletinko and Uskov [9] obtained empirical formulas for the frequency and amplitude of pressure oscillations in a cavity for jets with parameters $M_{atm} = 2.0$–$3.6$ and $n = 0.5$–$2.0$ and various cavities with $d_2/d_1 = 1$–$2$ and $h/d_2 = 0$–$10$. For $M_{atm} = 3.6$, $n = 1$, and $d_2/d_1 = 1$, the region of existence of auto-oscillations is in the range of $0.5L \leqslant l \leqslant 1.3L$.

In the present work, results are obtained for $M_{atm} = 1$ and values of $n > 1$ typical for the jets used for sound generation by means of the Hartmann effect. A formula is proposed for an oscillation frequency that gives a better fit to experiment than that obtained in [9]. A theoretical explanation is given for the experimentally determined range of existence of auto-oscillations.

In formulating a criterion for determining the region of existence of auto-oscillations, we are based on the same hypothesis as the authors of the papers cited above but we introduce two more assumptions.

Assumption 1. With continuous increase in $l$ within the first “barrel” of the jet, auto-oscillations arise when $x_m$ reaches the maximum point on the curve $p_0'(x)$ (see Fig. 2). With further increase in the distance from the nozzle to the resonator, the Mach disk and the reflected shock are in the region with a positive pressure gradient $p_0'$, i.e., in the region of jet instability.

Assumption 2. In the first “barrel,” $x_{max} \neq 0.5L$ and is calculated from formula (2).

Under these assumptions, we lay down a rule for calculation of the minimum distance $l_{min}$ from the nozzle to the resonator at which auto-oscillations begin. We base on the well-known experimental results of [10] for the interaction of a supersonic underexpanded jet with a flat plate (obstacle) placed perpendicular to the jet axis.

According to [10], when the obstacle is placed in the first “barrel” of the jet, the distance from the nozzle to the Mach disk $x_m$ is calculated from the formula

$$x_m/x_{sh} = 1 - A\exp(-Bl/x_{sh}). \tag{3}$$

For an obstacle of large diameter ($d_{obst} \gg d_1$, where $d_1 = 2r_1$), Semiletinko and Uskov [10] propose values of $A = 1.13$ and $B = 1.36$. For small $d_{obst} > d_1$, formula (3) remains valid [11] but the values of $A$ and $B$ depend on the ratio $d_{obst}/d_1$.

Identifying the open end of the resonator tube with a flat rigid plate, assuming that $x_m = x_{max}$ in (3), and taking into account (2), we obtain

$$\frac{l_{min}}{L} = \frac{1}{B}\,\frac{x_{sh}}{L}\,\ln\frac{A}{1 - x_{max}/x_{sh}} = \frac{0.8}{B}\,\ln 5A. \tag{4}$$

To determine the values of $l_{min}$ and the validity of formula (4), we performed experiments for various gas-dynamic and geometrical characteristics of a Hartmann generator. The experiments were performed in a 2 m long channel whose cross section was a square 200 mm on side. A plane traveling wave propagated in the channel from the source of sound to the channel exit section. The frequency and intensity of the sound were determined by an LKh-610 piezoelectric transducer, an S5-3 spectrum analyzer, and an oscillograph. The walls of the resonator were 2 and 2.5 mm thick. Table 1 gives gas-dynamic and geometrical characteristics of a Hartmann generator calculated from formula (4) and experimental values of $l_{min}/L$ (the data in the last row are taken from [12]).

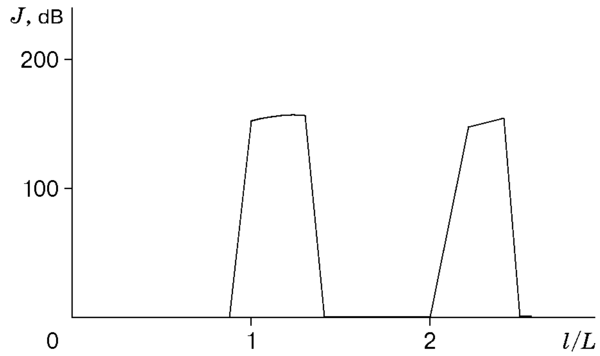


Fig. 3

An analysis of the experimental results shows the following:

— for the indicated gas-dynamic and geometrical parameters, the system of shock waves ahead of the resonator leads to a considerable increase in $l_{\min}$ compared to the data of [1];

— formula (4) can be used to estimate $l_{\min}$.

The upper bound of the region of existence of auto-oscillations is denoted by $l_{\max}$. According to (3), for $l = L$ (boundary of the first and second "barrels" of the jet), we have $x_{\rm m}/x_{\rm sh} < 1$, i.e., the system of shock waves in the first "barrel" of the jet is displaced toward the nozzle compared to its position in the free jet. According to Assumption 1 formulated above, auto-oscillations exist for $l = L$. Hence, unlike in [1], $l_{\max} > L$.

We take into account that for $x = L$ (see Fig. 2), the flow is similar to the flow in the nozzle exit section but with a smaller value of the total pressure $p_0'$, which is due to losses in the system of shock waves. When the resonator moves into the region $x > L$, a system of shock waves forms ahead of the resonator again because the flow here is supersonic. The configuration of this system is similar to the configuration for the first "barrel" of the jet because the transverse distribution of gas-dynamic parameters is similar to their distribution in the first "barrel."

Taking into account that the flow is stationary (auto-oscillations cease) when the indicated system of shock waves is located on the descending branch of the curve of $p_0'(x)$ of the second "barrel" ($L < l < 1.5L$), we obtain the position of the resonator at which this occurs. Only the smallest theoretical value of $l_{\max}$ can be obtained by simple means. We assume that the system of shock waves ahead of the resonator is in the section $x = L$ and determine the corresponding position of the resonator. For this, in formula (3), it suffices to set $x_{\rm m} = 0$ and calculate the values of $\Delta l$ for known $A$, $B$, and $x_{\rm sh}$. We obtain $l_{\max} = L + \Delta l$. This is the lower bound of $l_{\max}$, and its real values are somewhat larger. Values of $l_{\max}$ measured in experiments and calculated by the above-mentioned rule are presented in Table 1.

From the above results it follows that for $\mathrm{M_{atm}} = 1$, auto-oscillations exist in the range of $L < l < 1.5L$ for the first "barrel" of the jet and in the range of $2L < l < 2.5L$ for the second barrel. Figure 3 shows the dependence of sound intensity $J$ on the distance $l/L$ for the data given in the second row of Table 1.

In determining the oscillation frequency, Borisov [1], working with short resonators ($d_2 \sim h$), allowed for the distance between the resonator and the detached shock wave, including it in the resonator length. It can be stated *a priori* that for $h \gg d_2$, this is not required. In this case, the frequency $f$ should be calculated from the well-known formula of acoustics

$$f = \frac{c}{4(h + 0.3d_2)}. \qquad (5)$$

The frequencies calculated from formula (5) (velocity of sound $c = 340$ m/sec and $h = 170$ mm) for $d_2 = 14$ and 20 mm are equal to 485 and 479 Hz, respectively. For the same values of $c$, $h$, and $d_2 = 14$ mm, the experimental value of the frequency is equal to 455 Hz, and for $d_2 = 20$ mm, experiments give values of 450 and 440 Hz. The calculated data are in good agreement with experiment. The experimental values are slightly smaller than the calculated data because the latter are obtained on the basis of linear acoustics, and in the experiment, only intense, substantially nonlinear oscillations were studied.

## REFERENCES


1. Yu. Ya. Borisov, *Gas-Jet Oscillators of the Hartmann Type. Sources of Powerful Ultrasound*, L. D. Rosenberg (ed.) [in Russian], Nauka, Moscow (1967).
2. V. G. Dulov and G. A. Luk'yanov, *Gasdynamics of Flow Processes* [in Russian], Nauka, Novosibirsk (1984).
3. Tsein Che-Sun, "Axisymmetric supersonic turbulent jet issuing from a nozzle with underexpansion," in: G. N. Abramovich (ed.), *Investigation of Turbulent Air Jets, Plasma, and Real Gas* (collected scientific papers) [Russian translation], Mashinostroenie, Moscow (1967).
4. C. H. Lewis (Jr.) and D. J. Carlson, "Normal shock location in underexpanded gas and gas-particle jets," *AIAA J.*, **2**, No. 4 (1964).
5. Yu. V. Finat'ev and L. A. Shcherbakov, "On the possibility of approximating the boundaries of an underexpanded axisymmetric jet by an arc of an ellipse," *Inzh.-Fiz. Zh.*, **17**, No. 4, 737–741 (1969).
6. E. A. Ugryumov, "Gasdynamics of interaction of a supersonic jet with a dead-end channel," in: V. G. Dulov (ed.), *Gas-Dynamics and Acoustics of Jet Flows* (collected scientific papers) [in Russian], Inst. Theor. and Appl. Mech., Sib. Div., Acad. of Sci. of the USSR, Novosibirsk (1987), pp. 66–73.
7. G. V. Naberezhnova and Yu. N. Nesterov, "Unstable interaction of a supersonic underexpanded jet with a cylindrical cavity," *Uch. Zap. TsAGI*, **14**, No. 5, 58–64 (1983).
8. V. M. Kuptsov, S. I. Ostroukhova, and K. N. Filippov, "Pressure oscillations and heating of gas with flow of a supersonic jet into a cylindrical cavity," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 5, 104–111 (1977).
9. V. M. Ustinov, "Experimental investigation of pressure oscillations in a resonant tube," in: *Applied Problems of Hydrogasdynamics* (collected scientific papers) [in Russian], Regional Pedagog. Inst., Moscow (1986), pp. 52–60.
10. B. G. Semiletinko and V. N. Uskov, "Experimental dependences for the position of shock waves in a jet incident on an obstacle normal to the jet axis," *Inzh.-Fiz. Zh.*, **23**, No. 3, 453-458 (1972).
11. A. V. Solotchin, "Experimental investigation of the incidence of a supersonic underexpanded jet on a flat obstacle," Candidate's Dissertation in Tech. Sci., Novosibirsk (1982).
12. V. N. Glaznev, A. V. Solotchin, and Sh. Suleimanov, "Parametric analysis of auto-oscillations in the case of a supersonic jet flowing into a cylindrical cavity," *Izv. Sib. Otd. Aakd. Nauk SSSR, Ser. Tekh. Nauk*, No. 13, Issue 3, 22–26 (1983).